

# Simplified Description of the Field Distribution in Finlines and Ridge Waveguides and Its Application to the Analysis of *E*-Plane Discontinuities

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**Abstract**—Using closed-form equations for the field distribution of the eigenmodes in ridge waveguides, this paper presents a simplified analysis for ridge waveguide *E*-plane discontinuities. The accuracy of the calculated results is checked by comparison with experimental results. Closed-form equations are also presented for the field distribution of the dominant hybrid mode in unilateral and bilateral finlines. The usefulness of these equations in calculating the characteristic impedance and in determining the plane of the circularly polarized magnetic field in unilateral finlines is demonstrated.

## I. INTRODUCTION

FINLINE and ridge waveguide circuits have frequently been used in the design of microwave and millimeter-wave components that require single-mode broad-band operation. Such components usually incorporate different types of *E*-plane discontinuities. Over the past several years, various approaches [1]–[4] have been reported for the characterization of finline discontinuities. However, the great majority of these approaches were developed to treat structures with infinitely thin metallization thickness. Thus, they are not applicable to the analysis of ridge waveguide discontinuities.

More recently, two rigorous approaches have been reported for the characterization of ridge waveguide *E*-plane discontinuities [5], [6]. The first one [5] uses the spectral-domain technique and has the advantage of high numerical efficiency. It is, however, limited to structures with infinitely thin ridges. The second approach [6] applies the mode-matching technique and requires the determination of the field distribution of the ridge waveguide eigenmodes. In this approach, however, the field distribution of the TE and TM modes in the ridge waveguide is obtained by following the conventional method of solving the boundary value problem first for the eigenvalue (the propagation constant), then for the associated eigenvector. Be-

sides its complexity, the computational effort involved in this approach is extremely large.

On the other hand, approximate closed-form equations for the field distribution of the TE modes in ridge waveguides have been reported in [7]. These equations have been also used with the variational technique in [8] to analyze slot resonators. However, due to the coupling between TE and TM modes, formulation of the scattering matrix of the ridge waveguide discontinuity requires a knowledge of the field distribution of both TE and TM modes.

In this paper, we present closed-form equations for the field distribution of the TM modes in ridge waveguides. The closed-form equations for both TE and TM modes are then used with the conservation of complex power technique [9] to provide an efficient analysis of ridge waveguide *E*-plane discontinuities. The validity of this analysis is verified by comparing our results with published results as well as experimental results.

Although numerous closed-form equations have been reported in the literature [10], [11] for calculating the propagation constant in finlines, no paper has appeared for quick and easy evaluation of the field distribution. In this paper, we also present closed-form equations for the field distribution for the dominant mode in unilateral and bilateral finlines. In addition to the practical usefulness of these equations in the design of nonreciprocal finline components [12], these equations can also be used to derive a closed-form expression for the characteristic impedance.

## II. FIELD DESCRIPTION IN RIDGE WAVEGUIDES

The major complexity in formulating the scattering matrix of the discontinuity shown in Fig. 1 is in determining the field distribution of the eigenmodes in the ridge waveguide region. The propagation constants of the eigenmodes in ridge waveguides are related to the cutoff frequencies, which can be easily calculated according to [9]. With the assumption of a single term in the ridge gap (region II) of Fig. 2 and *N* expansion terms in the trough region (region I), and with the application of the boundary conditions on

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the tangential fields at  $x = l$ , the field distribution of the TE and TM modes can be written as

TE modes:

$$\vec{E}_I = \sum_{n=0,1,2,\dots}^N A_n \left[ \frac{n\pi}{b} \cos \alpha_n x \sin \frac{n\pi}{b} y \vec{a}_x \right. \\ \left. - \alpha_n \sin \alpha_n x \cos \frac{n\pi}{b} y \vec{a}_y \right] e^{-j\beta z} \quad (1a)$$

$$\vec{H}_I = \sum_{n=0,1,2,\dots}^N A_n \frac{\beta}{\omega\mu} \left[ \alpha_n \sin \alpha_n x \cos \frac{n\pi}{b} y \vec{a}_x \right. \\ \left. + \frac{n\pi}{b} \cos \alpha_n x \sin \frac{n\pi}{b} y \vec{a}_y \right. \\ \left. + \frac{k_c^2}{j\beta} \cos \alpha_n x \cos \frac{n\pi}{b} y \vec{a}_z \right] e^{-j\beta z} \quad (1b)$$

$$\vec{E}_{II} = \cos k_c \left( \frac{a}{2} - x \right) \vec{a}_y e^{-j\beta z} \quad (1c)$$

$$\vec{H}_{II} = -\frac{\beta}{\omega\mu} \left[ \cos k_c \left( \frac{a}{2} - x \right) \vec{a}_x \right. \\ \left. + \frac{k_c}{j\beta} \sin k_c \left( \frac{a}{2} - x \right) \vec{a}_z \right] e^{-j\beta z} \quad (1d)$$

where

$$A_n = \frac{-\Gamma_n \cos(k_c s/2)}{\alpha_n \sin(\alpha_n l)(n\pi)} \left[ \sin \frac{n\pi}{b} (h+d) - \sin \frac{n\pi}{b} h \right] \quad (1e)$$

$\Gamma_n = 1$  for  $n = 0$  and  $\Gamma_n = 2$  for  $n = 1, 2, 3, \dots$ .

TM modes:

$$\vec{E}_I = \sum_{n=1,2,3,\dots}^N B_n \left[ \alpha_n \cos \alpha_n x \sin \frac{n\pi}{b} y \vec{a}_x \right. \\ \left. + \frac{n\pi}{b} \sin \alpha_n x \cos \frac{n\pi}{b} y \vec{a}_y \right. \\ \left. - \frac{k_c^2}{j\beta} \sin \alpha_n x \sin \frac{n\pi}{b} y \vec{a}_z \right] e^{-j\beta z} \quad (2a)$$

$$\vec{H}_I = \sum_{n=1,2,3,\dots}^N B_n \frac{\omega\epsilon}{\beta} \left[ -\frac{n\pi}{b} \sin \alpha_n x \cos \frac{n\pi}{b} y \vec{a}_x \right. \\ \left. + \alpha_n \cos \alpha_n x \sin \frac{n\pi}{b} y \vec{a}_y \right] e^{-j\beta z} \quad (2b)$$

$$\vec{E}_{II} = \left[ \gamma \sin \gamma \left( \frac{a}{2} - x \right) \sin \frac{\pi}{d} (y-h) \vec{a}_x \right. \\ \left. + \frac{\pi}{d} \cos \gamma \left( \frac{a}{2} - x \right) \cos \frac{\pi}{d} (y-h) \vec{a}_y \right. \\ \left. - \frac{k_c^2}{j\beta} \cos \gamma \left( \frac{a}{2} - x \right) \sin \frac{\pi}{d} (y-h) \vec{a}_z \right] e^{-j\beta z} \quad (2c)$$

$$\vec{H}_{II} = \frac{\omega\epsilon}{\beta} \left[ -\frac{\pi}{d} \cos \gamma \left( \frac{a}{2} - x \right) \cos \frac{\pi}{d} (y-h) \vec{a}_x \right. \\ \left. + \gamma \sin \gamma \left( \frac{a}{2} - x \right) \sin \frac{\pi}{d} (y-h) \vec{a}_y \right] e^{-j\beta z} \quad (2d)$$

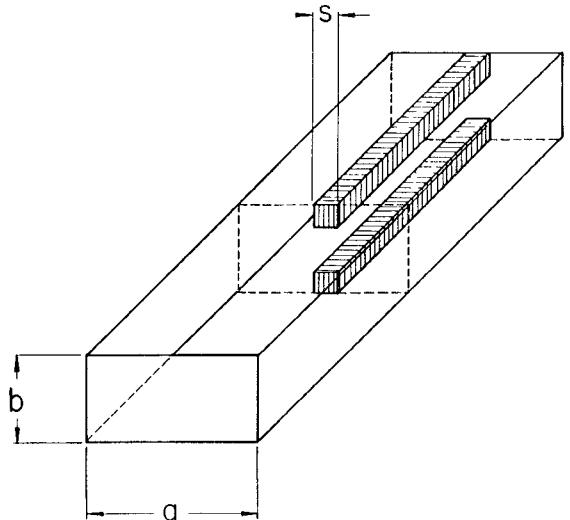


Fig. 1. Ridge waveguide  $E$ -plane discontinuity.

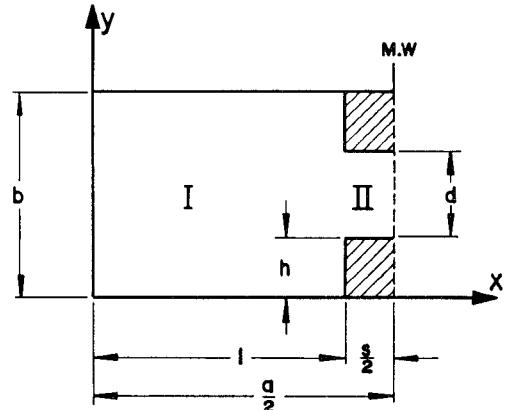


Fig. 2. Ridge waveguide with magnetic wall symmetry.

where

$$B_n = \frac{(\pi/d)2 \cos(\gamma s/2)}{b \sin \alpha_n l \left[ (\pi/d)^2 - (n\pi/b)^2 \right]} \cdot \left[ \sin \frac{n\pi}{b} (h+d) + \sin \frac{n\pi}{b} h \right]. \quad (2e)$$

The quantities  $\beta$ ,  $k_c$  and  $\gamma$  are related as

$$\beta^2 = k_0^2 - k_c^2 \quad k_c = 2\pi/\lambda_c$$

$$\alpha_n^2 = k_c^2 - \left( \frac{n\pi}{b} \right)^2 \quad \gamma^2 = k_c^2 - \left( \frac{\pi}{d} \right)^2.$$

Equations (1) and (2) not only give the field distribution of the dominant TE mode and dominant TM mode. They also can be used to represent that of the higher order modes. It should, however, be mentioned that due to the use of a single expansion term (i.e., one mode) in the ridge gap, these equations may not accurately describe the fields near the edge of the ridge, but they provide a reasonable description elsewhere. The validity of (1) and (2) will be verified in Section III. A simple check of the validity of this procedure can, however, be achieved by comparing the ridge waveguide impedance calculated from these equa-

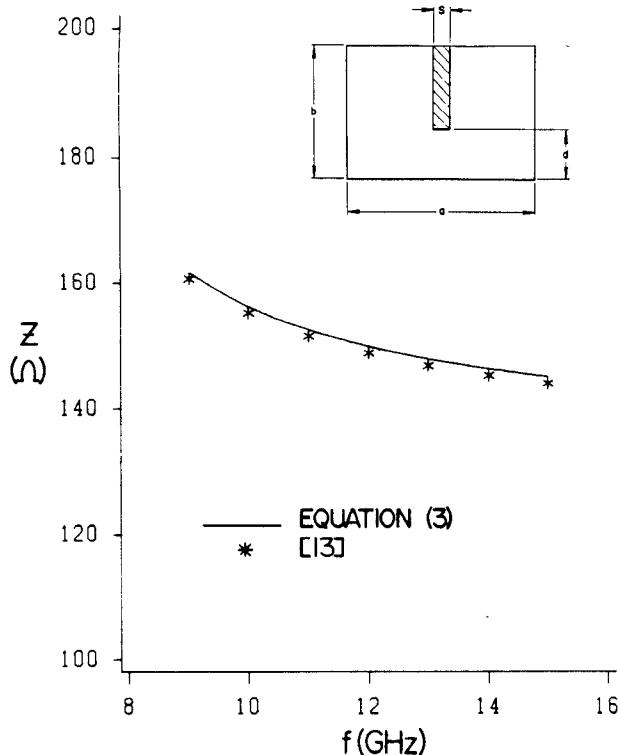


Fig. 3. Characteristic impedance versus frequency in ridge waveguide:  $a = 19.0$  mm,  $b = 8.55$  mm,  $s = 0.3$  mm,  $d = 1.7$  mm.

tions with other available data. The dominant mode characteristic impedance may be defined as

$$Z = \frac{V^2}{2P} = \frac{d^2}{2P} \quad (3a)$$

where  $V$  is the voltage across the ridge gap,  $d$  is the gap width, and  $P$  is the power propagation in the  $z$  direction, which can be written as

$$P = \frac{1}{2} \operatorname{Re} \int \vec{E} \times \vec{H}^* \cdot \vec{a}_z ds. \quad (3b)$$

Substituting (1) into (3b) gives

$$P = \frac{\beta}{2\omega\mu} \left[ \sum_{n=1,2,3}^N |A_n|^2 \left( \frac{n\pi}{b} \right)^2 \frac{b}{\Gamma_n} \left[ \frac{l}{2} + \frac{\sin 2\alpha_n l}{4\alpha_n} \right] \right. \\ \left. + \sum_{n=0,1,2,\dots}^N |A_n|^2 |\alpha_n|^2 \frac{b}{\Gamma_n} \left[ \frac{l}{2} - \frac{\sin 2\alpha_n l}{4\alpha_n} \right] \right. \\ \left. + d \left[ \frac{S}{4} - \frac{\sin k_c S}{4k_c} \right] \right]. \quad (3c)$$

Fig. 3 illustrates a comparison between the characteristic impedance calculated from the closed-form expression given in (3) and that obtained using the variational technique [13]. It is noted that there is a good agreement.

### III. ANALYSIS OF $E$ -PLANE RIDGE WAVEGUIDE DISCONTINUITIES

The ridge waveguide discontinuity shown in Fig. 1 represents the basic building block in the design of many microwave and millimeter-wave components, among them

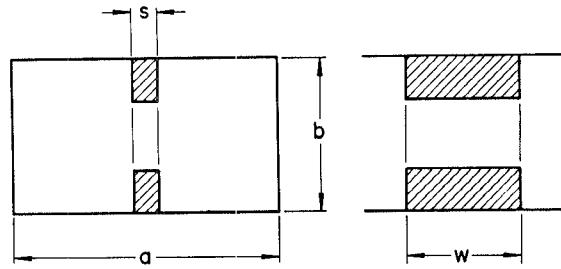


Fig. 4. Cascaded ridge waveguide  $E$ -plane discontinuity.

transformers [6] and evanescent mode filters [14]. Having determined the field distribution of the eigenmodes in ridge waveguides, the conservation of complex power technique [9] can then be employed to evaluate the scattering matrix of the ridge waveguide discontinuity. In view of [9], the scattering parameters may be written as

$$S_{22} = (\mathbf{P}_2^\dagger + \mathbf{H}^\dagger \mathbf{P}_1^\dagger \mathbf{H})^{-1} (\mathbf{P}_2^\dagger - \mathbf{H}^\dagger \mathbf{P}_1^\dagger \mathbf{H}) \quad (4a)$$

$$S_{12} = \mathbf{H}(\mathbf{I} + S_{22}) \quad (4b)$$

$$S_{21} = 2(\mathbf{P}_2^\dagger + \mathbf{H}^\dagger \mathbf{P}_1^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{P}_1^\dagger \quad (4c)$$

$$S_{11} = \mathbf{H}S_{21} - \mathbf{I} \quad (4d)$$

where  $\dagger$  denotes Hermitian transpose.  $\mathbf{H}$  is the  $E$ -field mode-matching matrix, which can be divided into four submatrices:

$$\mathbf{H} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}. \quad (5)$$

The submatrices  $\mathbf{A}$  and  $\mathbf{D}$  give the coupling between the TE and TM modes in the rectangular waveguide and the hybrid TE and TM modes in the ridge waveguide, whereas  $\mathbf{B}$  and  $\mathbf{C}$  indicate the cross-coupling between the modes in the two guides.  $\mathbf{P}_1$  and  $\mathbf{P}_2$  are diagonal matrices whose diagonal elements represent the power carried by unit amplitude modes in the rectangular and ridge waveguide, respectively. With the use of the generalized matrix technique [9], the overall scattering matrix of the cascaded ridge waveguide discontinuity shown in Fig. 4 can easily be evaluated.

In order to check the validity of this analysis, we compare in Fig. 5 our results with those reported in [5] using the spectral-domain technique. A good agreement is observed. Fig. 6 also shows a comparison between our results and experimental results for a structure with a ridge of finite metallization thickness. It is noted that there is a good agreement between the computed and the experimental results. The numerical results shown in Figs. 5 and 6 were obtained using (20 TE modes + 10 TM modes) in the rectangular waveguide and (eight TE modes + four TM modes) in the ridge waveguide.

In contrast to the variational technique used in [15], the effect of the higher order mode coupling can be taken into account in the present analysis. This in turn allows cascaded discontinuities to be accurately analyzed. Fig. 7 shows the transmission coefficient of two  $E$ -plane discontinuities connected in cascade. It is observed that the calculated results agree with the measured results.

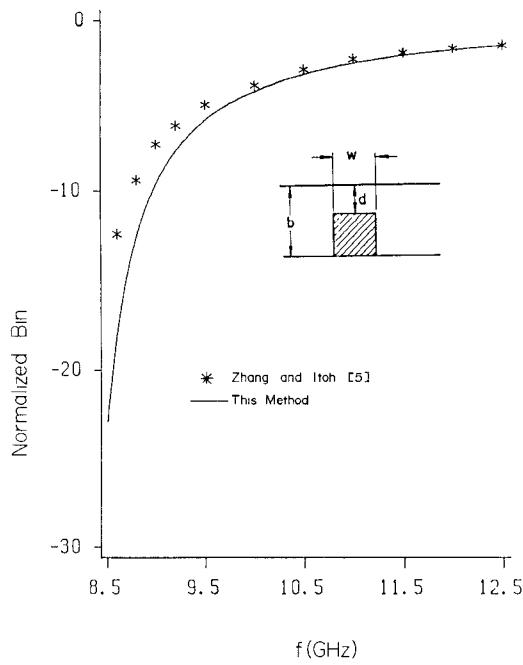


Fig. 5. Normalized susceptance of an *E*-plane ridge waveguide discontinuity:  $a = 22.86$  mm,  $b = 10.16$  mm,  $h = 0.0$ ,  $S = 0.0$ ,  $l = 11.43$  mm,  $d = 2.79$  mm,  $w = 1.7$  mm.

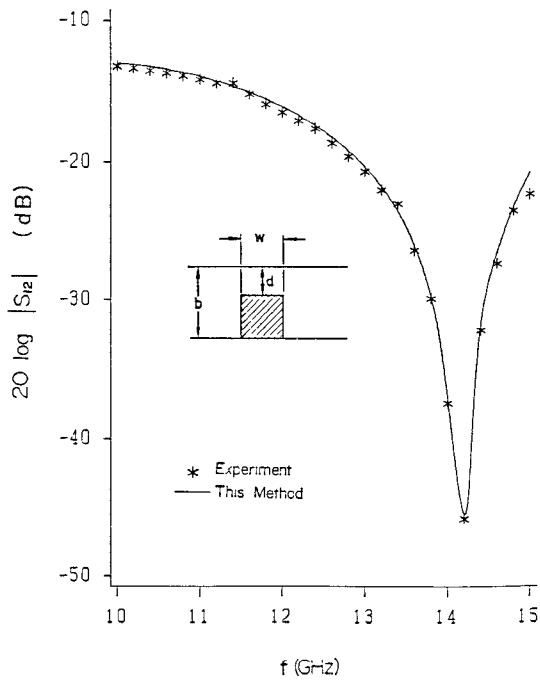


Fig. 6. Magnitude of transmission coefficient of an *E*-plane ridge waveguide discontinuity:  $a = 19.05$  mm,  $b = 9.524$  mm,  $h = 0.0$ ,  $S = 1.016$  mm,  $l = 9.017$  mm,  $d = 1.905$  mm,  $w = 5.08$  mm.

#### IV. FIELD DESCRIPTION IN UNILATERAL FINLINES

Due to the hybrid nature of the electromagnetic field in unilateral finlines, the field distribution is expressed as a summation of LSE and LSM modes. The propagation constant can be calculated using the closed-form expressions given in [10]. To demonstrate how the amplitudes of the LSE and LSM modes are determined, consider the

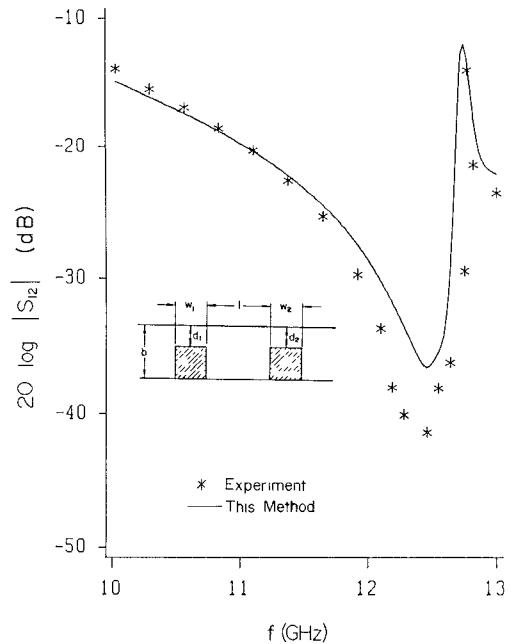


Fig. 7. Magnitude of transmission coefficient of a cascaded *E*-plane ridge waveguide discontinuity:  $a = 22.86$  mm,  $b = 10.16$  mm,  $h = 0.0$ ,  $S = 2.057$  mm,  $l = 10.40$  mm,  $d_1 = d_2 = 4.114$  mm,  $l' = 12.192$  mm,  $w_1 = w_2 = 1.524$ .

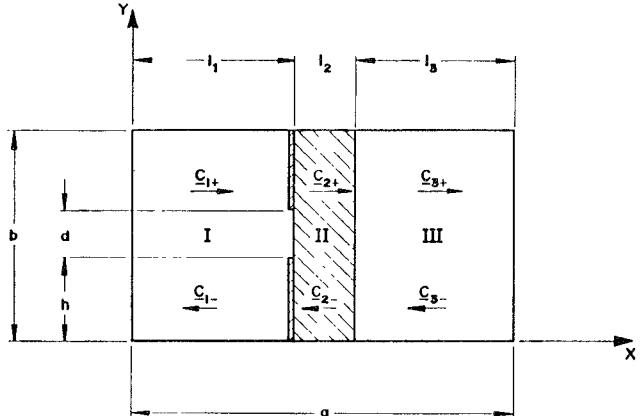


Fig. 8. A unilateral finline structure.

unilateral finline structure shown in Fig. 8. Let the incident and reflected *E*-field mode amplitude vectors in regions I, II, and III be respectively presented by  $\underline{C}_{1+}$  and  $\underline{C}_{1-}$  ( $i = 1, 2$  and  $3$ ). With the assumption of a constant field in the fin gap, continuity of the tangential field at  $x_1 = l_1$  and  $x_2 = l_1 + l_2$  gives

$$[\underline{C}_{1+}]_{x_1} + [\underline{C}_{1-}]_{x_1} = \mathbf{M} \quad (6a)$$

$$[\underline{C}_{2+}]_{x_1} + [\underline{C}_{2-}]_{x_1} = \mathbf{M} \quad (6b)$$

$$[\underline{C}_{2+}]_{x_2} + [\underline{C}_{2-}]_{x_2} = [[\underline{C}_{3+}]_{x_2} + [\underline{C}_{3-}]_{x_2}] \quad (6c)$$

where  $\mathbf{M}$  is a column matrix whose elements represent the coupling between the modes in region I and the constant field in the fin gap. The amplitude vectors  $\underline{C}_{i+}$  and  $\underline{C}_{i-}$

are also related by

$$[\underline{C}_{1+}]_{x_1} = -\mathbf{L}_1 \mathbf{L}_1 [\underline{C}_{1-}]_{x_1} \quad (7a)$$

$$[\underline{C}_{2+}]_{x_2} = \mathbf{L}_2 [\underline{C}_{2+}]_{x_1} \quad (7b)$$

$$[\underline{C}_{2-}]_{x_1} = \mathbf{L}_2 [\underline{C}_{2-}]_{x_2} \quad (7c)$$

$$[\underline{C}_{2-}]_{x_2} = \mathbf{S} [\underline{C}_{2+}]_{x_2} \quad (7d)$$

$$[\underline{C}_{3-}]_{x_2} = -\mathbf{L}_3 \mathbf{L}_3 [\underline{C}_{1+}]_{x_2} \quad (7e)$$

where  $\mathbf{L}_i$  ( $i=1, 2$ , and  $3$ ) are diagonal matrices with diagonal elements given by  $L_{i,nn} = e^{-j\alpha_n l_i}$ . The matrix  $\mathbf{S}$  is a diagonal matrix whose elements can be derived according to [16, ch. 6]. Manipulating (6) and (7) gives the amplitudes of the LSE and LSM modes in regions I, II, and III. The field distribution of the dominant mode can then be approximated as follows:

$$\vec{E} = \vec{E}^h + \vec{E}^e \quad \vec{H} = \vec{H}^h + \vec{H}^e$$

$$\vec{E}^h = \sum_{n=0,1,2,\dots}^N A_n \left[ j\beta \phi_n(x) \cos \frac{n\pi}{b} y \vec{a}_y \right. \\ \left. - \frac{n\pi}{b} \phi_n(x) \sin \frac{n\pi}{b} y \vec{a}_z \right] e^{-j\beta z} \quad (8a)$$

$$\vec{E}^e = \sum_{n=1,2,3,\dots}^N B_n \left[ \frac{K_{cn}^2}{j\alpha_n} \Psi_n(x) \sin \frac{n\pi}{b} y \vec{a}_x \right. \\ \left. + \frac{n\pi}{b} \phi_n(x) \cos \frac{n\pi}{b} y \vec{a}_y \right. \\ \left. - j\beta \phi_n(x) \sin \frac{n\pi}{b} y \vec{a}_z \right] e^{-j\beta z} \quad (8b)$$

$$\vec{H}^h = \sum_{n=1,2,3,\dots}^N A_n \frac{\alpha_n}{\omega \mu} \left[ \frac{K_{cn}^2}{j\alpha_n} \phi_n(x) \cos \frac{n\pi}{b} y \vec{a}_x \right. \\ \left. - \frac{n\pi}{b} \Psi_n(x) \sin \frac{n\pi}{b} y \vec{a}_y \right. \\ \left. - j\beta \Psi_n(x) \cos \frac{n\pi}{b} y \vec{a}_z \right] e^{-j\beta z} \quad (9a)$$

$$\vec{H}^e = \sum_{n=0,1,2,\dots}^N B_n \frac{\omega \epsilon}{\alpha_n} \left[ -j\beta \Psi_n(x) \sin \frac{n\pi}{b} y \vec{a}_y \right. \\ \left. - \frac{n\pi}{b} \Psi_n(x) \cos \frac{n\pi}{b} y \vec{a}_z \right] e^{-j\beta z} \quad (9b)$$

where  $A_n$ ,  $B_n$ ,  $\phi_n$ , and  $\Psi_n$  are below for the various regions.

#### Region I (Unilateral Finline)

$$\phi_n(x) = j \sin \alpha_{1n} x \quad \Psi_n(x) = \cos \alpha_{1n} x$$

$$A_n = \frac{-2j\beta W_n}{[e^{+j\alpha_{1n} l_1} - e^{-j\alpha_{1n} l_1}]} \quad B_n = \frac{2(n\pi/b) W_n}{[e^{+j\alpha_{1n} l_1} - e^{-j\alpha_{1n} l_1}]}$$

$$W_n = \frac{\Gamma_n}{n\pi} \frac{1}{K_{cn}^2} \left[ \sin \frac{n\pi}{b} (h+d) - \sin \frac{n\pi}{b} h \right].$$

#### Region II (Unilateral Finline)

$$\phi_n(x) = [S_n e^{+j\alpha_{2n}(x-l_1)} + e^{-j\alpha_{2n}(x-l_1)}]$$

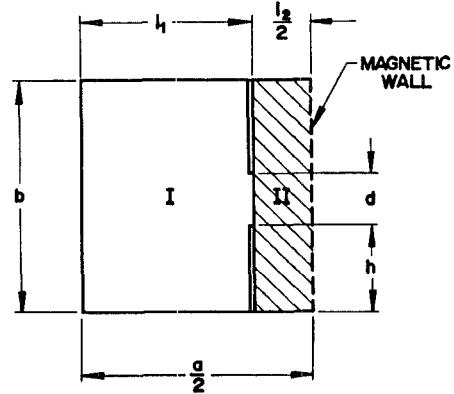


Fig. 9. A bilateral finline structure with magnetic wall symmetry.

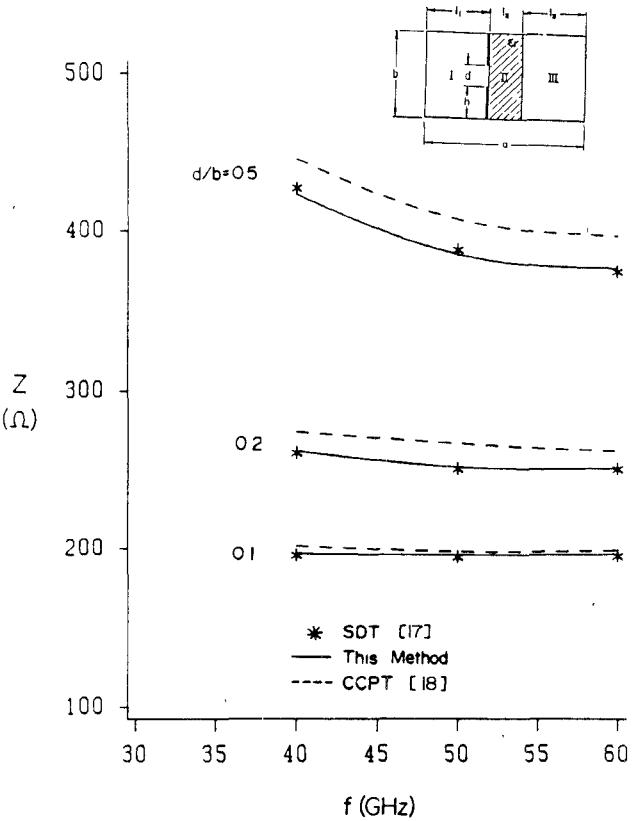


Fig. 10. Characteristic impedance versus frequency in unilateral finlines:  $a = 2b = 4.7752$  mm,  $l_2 = 0.127$  mm,  $l_1 = 2.3876$  mm,  $\epsilon_r = 2.2$ .

$$\Psi_n(x) = [S_n e^{+j\alpha_{2n}(x-l_1)} - e^{-j\alpha_{2n}(x-l_1)}]$$

$$A_n = -j\beta \frac{W_n}{(1+S_n)} \quad B_n = \frac{n\pi}{b} \frac{W_n}{(1+S_n)}$$

$$S_n = e^{-j2\alpha_{2n} l_2} \frac{(1-C_n) - (1+C_n) e^{-j2\alpha_{3n} l_3}}{(1+C_n) - (1-C_n) e^{-j2\alpha_{3n} l_3}}$$

$$C_n = \frac{\alpha_{2n}^*}{\alpha_{2n}^*} \quad \text{for LSE modes}$$

$$C_n = \frac{\alpha_{2n}^*}{\epsilon_r \alpha_{3n}^*} \quad \text{for LSM modes.}$$

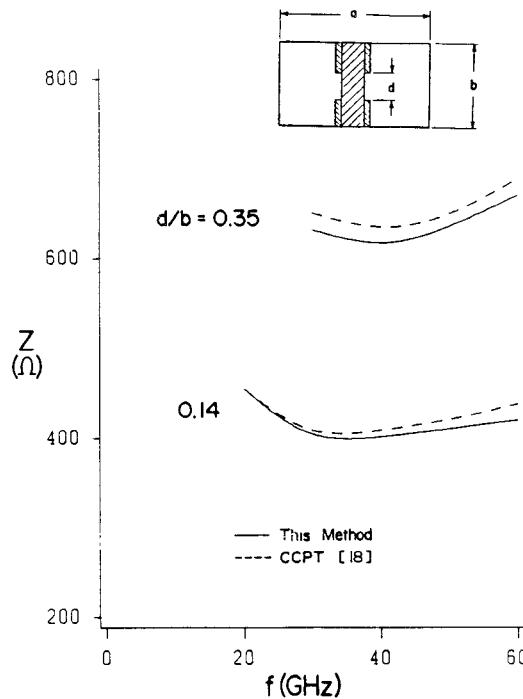


Fig. 11. Characteristic impedance versus frequency in bilateral finlines:  $a = 2b = 7.112$  mm,  $l_2 = 0.125$  mm,  $\epsilon_r = 3.0$ .

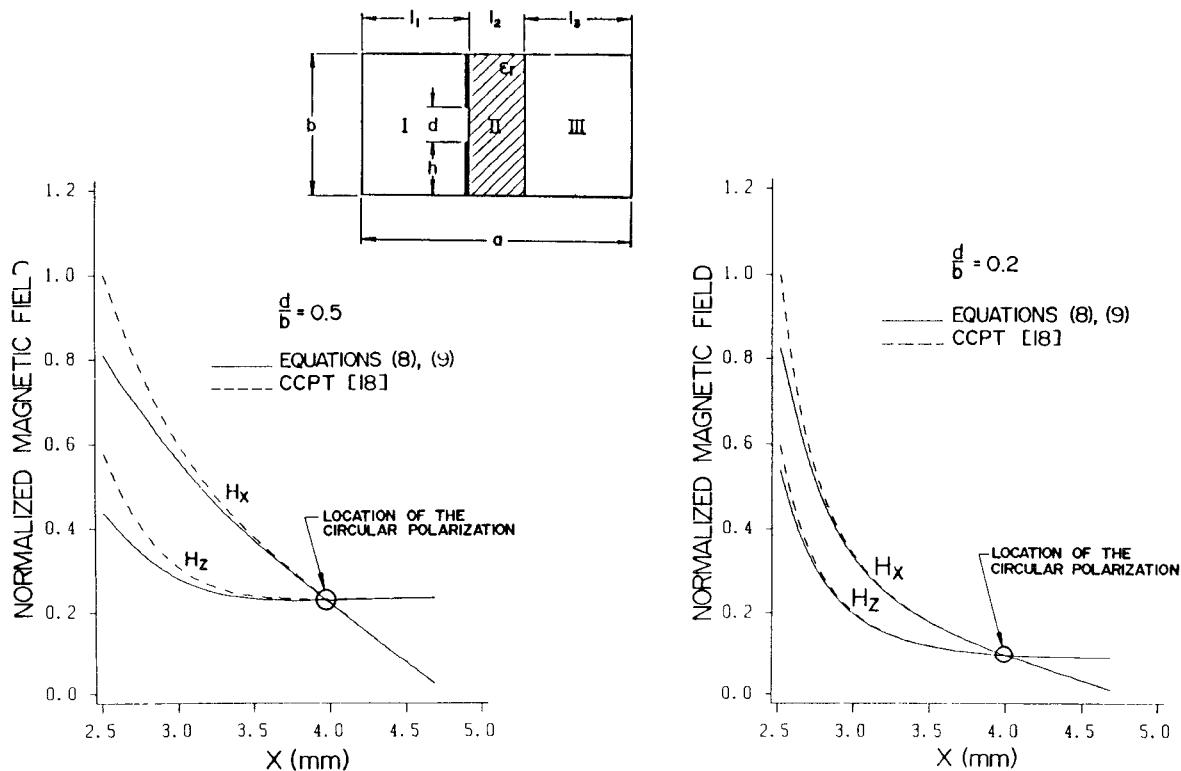


Fig. 12. The normalized magnetic field components in unilateral finlines:  $a = 2b = 4.7752$  mm,  $l_2 = 0.127$  mm,  $l_1 = 2.3876$  mm,  $\epsilon_r = 2.2$ .

### Region III (Unilateral Finline)

$$\begin{aligned}\phi_n(x) &= j \sin \alpha_{3n}(x - a) & \Psi_n(x) &= \cos \alpha_{3n}(x - a) \\ A_n &= -2j\beta W_n Y_n & B_n &= 2 \frac{n\pi}{b} W_n Y_n \\ Y_n &= \frac{[e^{-j\alpha_{2n}l_2} + e^{j\alpha_{2n}l_2} S_n]}{[e^{-j\alpha_{3n}l_3} - e^{+j\alpha_{3n}l_3}][1 + S_n]}\end{aligned}$$

where

$$\Gamma_n = 1 \text{ for } n = 0 \text{ and } \Gamma_n = 2 \text{ for } n = 1, 2, 3 \dots$$

$$K_{cn}^2 = \left(\frac{n\pi}{b}\right)^2 + \beta^2 \quad \alpha_{in}^2 = k_0^2 \epsilon_{ri} - K_{cn}^2.$$

For bilateral finlines, due to symmetry around the  $x$  axis we only need to consider regions I and II, as shown in Fig. 9. The field distribution in region I is similar to that described in region I of the unilateral finline structure shown in Fig. 8. In region II, however,  $A_n$ ,  $B_n$ ,  $\phi_n$  and  $\Psi_n$  are given as follows.

### Region II (Bilateral Finline)

$$\begin{aligned}\phi_n(x) &= \cos \alpha_n \left(x - \frac{a}{2}\right) & \Psi_n(x) &= j \sin \alpha_n \left(x - \frac{a}{2}\right) \\ A_n &= -j\beta W_n Y_n & B_n &= \frac{n\pi}{b} W_n Y_n \\ Y_n &= \frac{e^{-j\alpha_{2n}l_2/2}}{[1 + e^{-j\alpha_{2n}l_2}]}\end{aligned}$$

Having obtained the  $E$  and  $H$  field components, the characteristic impedance can be evaluated as demonstrated in Section II. In Fig. 10 we compare the characteristic impedance of unilateral finlines calculated using (8) and (9) with those reported in [17] using the spectral-domain technique. It is noted that there is a good agreement. It should, however, be mentioned that in [17] a constant field is also assumed in the fin gap. In order to verify the validity of (8) and (9), Fig. 10 also shows results calculated using the more exact analysis reported in [18]. Good agreement is observed when  $d/b$  is relatively small. Similar results were also obtained for bilateral finlines, as illustrated in Fig. 11.

Another useful application of (8) and (9) is in determining the plane of the pure circularly polarized magnetic field in unilateral finlines, which is needed in the design of finline isolators [12]. Fig. 12 shows a comparison between the normalized magnetic field components for a wave propagating in the positive  $z$  direction calculated using (8) and (9) and those obtained using the rigorous analysis [18]. The location of the plane of the circularly polarized magnetic field is also illustrated in this figure for the two cases of  $d/b = 0.2$  and  $d/b = 0.5$ .

It is concluded from Figs. 6, 7, 10, 11, and 12 that (1), (2), (8), and (9) can reasonably well approximate the field distribution in structures with relatively small gap sizes ( $d/b < 0.2$ ). This is, however, the case in most practical applications.

### V. CONCLUSIONS

Novel closed-form equations are reported for the field distribution of TM modes in ridge waveguides as well as the dominant mode in unilateral and bilateral finlines. Numerical results are presented which confirm the usefulness of these equations in calculating the characteristic impedance and determining the location of the circularly polarized magnetic field in unilateral finlines. The use of these equations in achieving a simplified analysis of  $E$ -plane ridge waveguide discontinuities has been demonstrated. The numerical results obtained for different discontinuities agree well with the experimental data.

### ACKNOWLEDGMENT

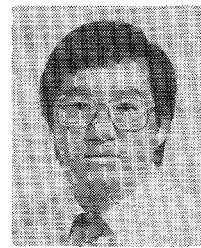
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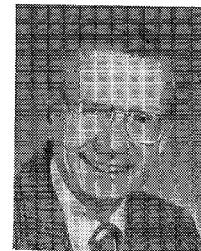
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